MATH 101, FALL 2018: SUM OF INTERIOR ANGLES OF POLYGONS

Theorem. If a polygon is drawn by picking $n \geq 3$ points on a circle and connecting them in consecutive order with line segments, then the sum of the interior angle of that polygon is $(n-2)180$ degrees.

Proof. We prove by induction on $n \geq 3$ the statement S_n: any polygon drawn by picking n points on a circle and connecting them in consecutive order with line segments has the sum of its interior angles equal to $(n-2)180$ degrees.

- Base case: if $n = 3$, then the statement says that the sum of the interior angles of a triangle is 180. This is a result of high school geometry (which we take for granted for that problem).
- Inductive step: Assume $n \geq 3$ and S_n is true. Assume $P_1, P_2, \ldots, P_n, P_{n+1}$ are $(n+1)$-many points on a circle (in consecutive order), and consider the polygon A made from connecting these points. Drawing a line from P_1 to P_3, we obtain a triangle T with points P_1, P_2, P_3, and another polygon B with points $P_1, P_3, P_4, \ldots, P_{n+1}$. Note that B has n points, so by the induction hypothesis the sum of its interior angles is $(n-2)180$. Also, T is a triangle so the sum of its interior angles is 180. Now observe that the sum of the interior angles of A is just the sum of the interior angles of B plus the sum of the interior angles of T. Therefore the sum of the interior angles of A is $(n-2)180 + 180 = ((n+1)-2)180$. Thus S_{n+1} is true.

By the principle of mathematical induction, S_n is true for all natural numbers $n \geq 3$.

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