MATH 123 - ALGEBRA II - SPRING 2020
ASSIGNMENT 7

Due Friday, March 27, 11h59pm. (please submit your assignment as a PDF on Canvas). Unless otherwise noted, references are to Dummit and Foote, *Algebra*, 3rd edition, Wiley, 2004.

Practice problems

(*Not* for credit. No need to submit your solutions. Try to do them while minimally looking at the textbook)

(1) Give the definition of the following: simple extension, algebraic extension, finitely generated field extension, composite of two fields.
(2) Prove that the minimal polynomial of an algebraic element over a given field is unique.
(3) Prove that if $F \subseteq K \subseteq L$ are field extensions, then $[L : F] = [K : F][L : K]$.
(4) Prove that the algebraic elements in a field extension themselves form a field.
(5) True or false?
  (a) $\sqrt{3} + \sqrt[3]{2} + i$ is algebraic (over $\mathbb{Q}$).
  (b) Any finite extension is algebraic.
  (c) Any algebraic extension is finite.

Problems for credit

(1) You should have the assignment from another student available for review in your Canvas todo list. Review problem 4 from that assignment (submit your comment on Canvas, *not* with this assignment). Refer to the peer review instructions on the course website for more details. *You are encouraged not to look at the official solution before submitting your review!*
(2) Prove or disprove:
  (a) e is algebraic over $\mathbb{Q}(\sqrt{2})$ *[you may use without proof that e is transcendental over $\mathbb{Q}$].*
  (b) Any simple extension is finite. *[Note: you may want to think about the converse. We will explore it in future classes].*
  (c) Any finite extension is finitely generated.
  (d) Any finitely generated extension is finite.
(3) (DF, 13.2.5) Let $F = \mathbb{Q}(i)$. Prove that $x^3 - 2$ and $x^3 - 3$ are irreducible over F.
(4) (DF, 13.2.7) Prove that $\mathbb{Q}(\sqrt{2} + \sqrt{3}) = \mathbb{Q}(\sqrt{2}, \sqrt{3})$ and find an irreducible polynomial over $\mathbb{Q}$ satisfied by $\sqrt{2} + \sqrt{3}$.
(5) (DF, 13.2.10) Determine the degree of the extension $\mathbb{Q}(\sqrt{3 + 2\sqrt{2}})$ over $\mathbb{Q}$.
(6) (DF, 13.2.16) Let K/F be an algebraic field extension and let R be a *subring* of K containing F. Show that R is a *subfield* of K. Is it still true if K/F is not an algebraic extension?
(7) (DF, 13.2.19) Let K/F be a field extension of degree n. Prove that K is isomorphic to a subfield of the ring of $n \times n$ matrices over F. *Hint: first show that for each fixed $\alpha \in K$, multiplication by α gives an F-linear transformation from K to K.*

Date: March 11, 2020.