MATH 123 - ALGEBRA II - SPRING 2020
ASSIGNMENT 10

Due Tuesday, April 14, 11h59pm. (please submit your assignment as a PDF on Canvas). Unless otherwise noted, references are to Dummit and Foote, *Algebra*, 3rd edition, Wiley, 2004.

Practice problems

(*Not* for credit. No need to submit your solutions. Try to do them while minimally looking at the textbook)

(1) State and prove the fundamental theorem of Galois theory.
(2) Prove that an extension K/F is Galois if and only if K is the splitting field of a separable polynomial in $F[x]$.
(3) Let K/F be an extension. True or false?
   (a) The fixed field of $\mathrm{Aut}(K/F)$ always contains F.
   (b) The fixed field of $\mathrm{Aut}(K/F)$ is always contained in F.
   (c) If K/F is a Galois extension and E is an intermediate field, then K/E is Galois.
   (d) If K/F is a Galois extension and E is an intermediate field, then E/F is Galois.

Problems for credit

(1) You should have the assignment from another student available for review in your Canvas todo list. Review problem 2 from that assignment (submit your comment on Canvas, *not* with this assignment). Refer to the peer review instructions on the course website for more details. *You are encouraged not to look at the official solution before submitting your review!*
(2) (DF, 14.2.1) Determine the minimal polynomial over $\mathbb{Q}$ for $\sqrt{2}+\sqrt{5}$.
(3) (DF, 14.2.3)
   (a) Determine the Galois group of the polynomial $f(x) = (x^2-2)(x^2-3)(x^2-5) \in \mathbb{Q}[x]$.
   (b) Describe *all* the subfields of the splitting field of $f(x)$.
(4) (DF, 14.2.5) Prove that the Galois group of $x^p - 2 \in \mathbb{Q}[x]$, for p a prime, is isomorphic to the group of matrices $\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}$, where $a, b \in \mathbb{F}_p$, $a \neq 0$.
(5) Prove that n divides $\phi(p^n-1)$ for any prime p and any integer $n \geq 1$. *Hint: study the automorphisms of the multiplicative group of* $\mathbb{F}_{p^n}$.

Date: April 6, 2020.