MATH 123 - ALGEBRA II, SPRING 2020

WF 9-10h15, Sever 214

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Course website: http://math.harvard.edu/~sebv/123-spring-2020/

Covid-19 update (3/10). The syllabus was updated to change the way the grade will be computed (the midterm was cancelled). Please ignore references to "in class" or physical places at Harvard: all instruction will take place remotely from now on. See the course website, or email me, for more details.

Course overview. The course will be an introduction to the theory of rings and fields. A *ring* is a set with two operations on it, addition and multiplication, satisfying the basic properties of these operations on the integers. A *field* is a ring with some additional properties, those satisfied by the rationals (nonzero elements have multiplicative inverses). We will cover topics such as the abstract theory of factorization in rings (generalizing the factorization of integers into primes), the theory of modules (they are the "ring actions": vector spaces are a particular case), and the theory of field extensions, including Galois theory (relating the structure of fields to group theory).

We will not only discuss the abstract theory: a major goal of the class is to see some applications of this algebraic machinery to classical problems, including the impossibility of performing several straightedge and compass constructions (trisecting an angle, doubling the cube, squaring the circle), as well as the famous insolvability of the quintic: there is no formula involving only additions, multiplications, and extracting roots, which gives the zeroes of a polynomial of degree five. The theory of finite fields is of much use also outside of mathematics, with applications in cryptography and other parts of computer science. This semester, there are classes at Harvard that discuss the strong connections with number theory (Math-129) and algebraic geometry (Math-137).

Course texts. The primary textbook for the class is:

> David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd edition, Wiley, 2004.

The book may be supplemented by online notes for selected topics. Some other *optional* references can be found on the course website (more will be added as class proceeds).

Prerequisites. A first course in abstract algebra, such as Math-122 or Math-55a is a must. For example, you should be comfortable with basic group theory (including, very importantly, the concept of a quotient). We will however start ring theory from scratch (but going fast at the beginning). You should also know linear algebra (at the level of 21a, or preferably higher). You *may* still be able to take the class if you do not have these exact prerequisites but are strongly motivated (for example, the group theory part of Math-101 may be sufficient instead of Math-122). Please talk to me to make sure.

Assessment. Your grade for the course will be determined by scores on homework assignments and one exam as follows:

- There will be one final exam, which will count for *35%* of your final grade.
- There will tentatively be 12 *homework assignments*. Cumulatively, they will count for *65%* of your final grade. Your two lowest homework scores will be dropped. See below for more details on homework assignments.

Due to the pandemic, it was announced 3/27 that all classes at Harvard this semester will be graded SEM/UEM (satisfactory emergency, unsatisfactory emergency). If your final class score is 70% or more, you will get SEM regardless of how the rest of the class performs. If your final score is slightly below 70%, you *may* still get SEM depending on factors such as how the rest of the class performed, your class participation, especially insightful discussions or assignment solutions, etc.

If you qualify for special accommodation (such as extra time) for the tests, or if you already know you will not be able to take one of the tests at the planned time (e.g. because of a religious observation or a university event), please let me know as soon as possible.

Exams. The tentative date for the final exam is Wednesday, April 29, at 10h15am. You will have 48 hours to solve it (you will be allowed to consult your own notes and the course book) and it will be due back on Friday, April 31, 10h15am.

Homework assignments. They will be announced in class and posted on the course website. Solutions will typically be posted the day after an assignment is due. Assignments will usually be due on Tuesday at midnight. Only partial credit will be given if an assignment is turned in late, and no credit if it is turned in after solutions have been distributed. Assignments have to be submitted *online*, via the course's Canvas site (good quality scans are acceptable, but using latex is encouraged).

Assignments will have several parts. One part will be for practice but not for credit: it will encourage you to remember arguments seen in class and ask yourself some basic questions. Another part will be for credit and consist of problems, of various kinds, using the material seen in class. You will also be asked to peer-review the answer of another student to a problem from a previous assignment. This means you will have to carefully read their answer and write one or two paragraphs on what you think is well done and what you think can be improved. More information about the peer review will be posted on the course website.

Assignments are one of the key elements of this course. You should make every effort to write down your thoughts clearly and precisely. Your writeup should contain little to no extraneous material (no scrap work). I also encourage you to *be intellectually honest*: it is better to say that you are not exactly sure how to solve a problem / justify a particular step and write your thoughts than to write three pages of obscure equations and hope readers will trust your solution to be correct (they will not).

On the first page of your assignment please include:

- Your full name.
- The *list of other students with whom you collaborated* (if any).

As long as you list your collaborators, *collaboration is allowed and encouraged*. You may discuss ideas on, and even possible solutions of, specific problems. *However*, you may *not* maintain a record (written, audio, photographic, etc.) of the discussion. This means that *you are required to write up solutions entirely on your own* and that you cannot show the assignment you are submitting to other students. For example, if you discuss a problem with others using a blackboard, you must erase the board once the discussion is over and write up the solution on your own.

Reading assignments. The reading assignments that are relevant for the lectures of any given week will be posted to the course website at

the end of the previous week. Some of the reading may not be discussed in class, and the class may also cover topics not in the reading. You will benefit from looking at the reading (if only for a few minutes) before coming to class. After reading, ask yourself the following questions: what are the key ideas/tools/methods introduced? What problem do they solve? How is this done, mathematically speaking? How would I have been able to think of this? When you read a theorem followed by a proof, ask yourself things like: is the hypothesis necessary? Is the converse true? Where does the proof use the hypothesis? In the words of Paul Halmos, "don't just read it, fight it!"

Resources for help. Many resources outside of class are offered, and I highly encourage you to take advantage of them:

- *Office hours (in my office, SC 321H)*: Office hours are times when I will be sitting in my office, just waiting for you to come ask me anything related to this class. You don't need to make an appointment to visit me during office hours; just come by! If you can't make it to my scheduled office hours, you are always welcome to email me, and we can set up another time to meet.

  I'll announce the times of my office hours soon, after I've had a chance to look over your schedules.

  Course assistant(s) will also hold office hours (to be announced).
- *Math Night*: Math Night will be held every Monday night, 8-10PM at the Leverett house dining hall. It is hoped that you will find students from Math 123 (or other math classes) there to work on problem sets or get help with them. "Extra special" food is promised.

Wellness and mental health. This class is meant to challenge you, not to pressure and overwork you to the point of making your life miserable and interfering with your health. If for any reason you feel the workload for this class is too much, come talk to me.

Throughout the semester, take care of yourself. Make it a top priority to maintain a healthy lifestyle by eating well, exercising regularly, avoiding drugs and excessive alcohol, getting enough sleep and taking some time to relax. This will help you achieve your goals and cope with stress.

All of us benefit from support during times of struggle. You are not alone. There are many helpful resources available on campus and an important part of the college experience is learning how to ask for help. Asking for support sooner rather than later is often helpful.

If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, I strongly encourage you to seek support. Counseling and Mental Health Services (CAMHS) is here to help: call 617-495-2042 or visit their website at `https://camhs.huhs.harvard.edu/find-help-now` Consider reaching out to a friend, faculty or family member you trust for help getting connected to the support that can help.

If you have questions about this or your coursework, please let me know.

Other policies.

Contacting me. Feel free to talk to me anytime. I will often be in my office, and you are welcome to drop by, but I might tell you I am busy if you come outside regular office hours. The best way to otherwise contact me is via email, as it provides me with a written record of our conversation.

Attendance. I strongly encourage you not only to attend lectures, but also to actively participate in them: stop me if anything is unclear and feel free to share your thoughts about the material (what do you find easy? What do you find hard?).

I expect your full attention during lectures: no loud conversation, use of cell phones, or other activities unrelated to class.

Grading issues. If you have any questions or complaints concerning the way an assignment has been graded that cannot be resolved with the CA, please come talk to me. You should first look at the official solution and make sure you understand it.

Academic integrity. Any acts of academic dishonesty, such as cheating, plagiarism, etc. will be dealt with according to University Policy. Examples of violation include searching the web (or inside a textbook) for solutions, copying part of another student's assignment or showing your assignment to another student. Please speak to me if you have any questions about this.