MATH 141A - MATHEMATICAL LOGIC I, FALL 2018

MWF 12PM-1h15PM, SC 310

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Course assistant: TBD

Course website: http://math.harvard.edu/~sebv/141a-fall-2018/

Course overview. The course is an introduction to mathematical logic. Roughly, mathematical logic studies the "building blocks" of mathematics (axioms, proofs, structures, sizes, etc.) as *mathematical objects themselves*. This leads to general and beautiful statements that are interesting of their own, but also can be applied to classical mathematical objects such as boolean algebras, graph, linear orders, fields, etc.

On the purely logical side, we will prove the *completeness theorem*, which says (roughly) that if a first-order statement is true, then it actually can be proven. A consequence is the *compactness theorem*, which says that if every finite subsets of a collection of first-order statements is true, then the entire collection is true. A recurring fundamental theme will be the *back and forth method* which studies how to build isomorphisms from small pieces. On the applied side, we will discuss for example why the rationals numbers with their usual ordering are the unique countable dense linear order without endpoints (and why this means there is an algorithm to decide the truth of any first-order statement about the rationals!), what nonstandard analysis is, why the chromatic number of the plane is witnessed by a finite graph, or why any injective polynomial mapping from $\mathbb{C}^n$ to $\mathbb{C}^n$ is also surjective (the Ax-Grothendieck theorem).

Course text. There is a single required textbook for this course which is *a course in model theory - an introduction to contemporary mathematical logic* by Bruno Poizat. It is published by Springer-Verlag and *is also available online for free to Harvard students* (see the course website).

Some other *optional* references are listed on the course website.

Prerequisites. This is a class in pure mathematics: I will assume that you are familiar with mathematical proofs. For this reason, a previous mathematics course at the level of Mathematics 23ab, 25ab, 55ab, 101, 102, or 112 is required. If you have not taken any of these classes or otherwise have doubts whether you satisfy the prerequisites for this course, please come talk to me.

Assessment. Your grade for the course will be determined by scores on homework assignments, a take home midterm exam, and a take home final exam as follows:

- The *take home midterm exam* will be handed out on October 15 and due two days later.
- The *take home final exam* will be handed out on December 3 and due two days later. Your highest score among the two exams will count for *40%* of your final grade, and the other will count for *30%*.
- There will tentatively be 12 *homework assignments*. Cumulatively, they will count for *30%* of your final grade. Your two lowest homework scores will be dropped. See below for more details on homework assignments.

The *highest* grade cutoffs will be as follows: 90%: A, 80%: B, 70%: C, 60%: D. These cutoffs *might* be lowered, but will not be raised. This means, for example, that you will be guaranteed an A if you score 90% or more, regardless how the rest of the class performs.

If you qualify for special accommodation (such as extra time) for the tests, or if you already know you will not be able to take one of the tests at the planned time (e.g. because of a religious observation or a university event), you should let me know as soon as possible.

Homework assignments. They will be announced in class and posted on the course website. Solutions will typically be posted the day after an assignment is due. Assignments will usually be due at the *beginning* of class. Only partial credit will be given if an assignment is turned in after class, and no credit if it is turned in after solutions have been distributed. Assignments have to be submitted *online*, via the course's Canvas site (scans are acceptable).

Assignments are one of the key elements of this course. You should make every effort to write down your thoughts clearly and precisely. Your writeup should contain little to no extraneous material (no scrap work). I also encourage you to *be intellectually honest*: it is better to say that you are not exactly sure how to solve a problem / justify a

particular step and write your thoughts than to write three pages of obscure equations and hope the grader will trust your solution to be correct.

On the first page of your assignment please include:

- Your full name.
- The *list of other students with whom you collaborated* (if any).

As long as you list your collaborators, collaboration is allowed and encouraged. You may discuss ideas on, and even possible solutions of, specific problems. *However*, you may *not* maintain a record (written, audio, photographic, etc.) of the discussion. This means that *you are required to write up solutions entirely on your own* and that you cannot show the assignment you are submitting to other students. For example, if you discuss a problem with others using a blackboard, you must erase the board once the discussion is over and write up the solution on your own.

Reading assignments. The reading assignments that are relevant for the lectures of any given week will be posted to the course website at the end of the previous week. You will benefit from looking at the reading before coming to class.

Resources for help. Many resources outside of class are offered, and I highly encourage you to take advantage of them:

- *Office hours (in my office, SC 321H)*: Office hours are times when I will be sitting in my office, just waiting for you to come ask me anything related to this class. You don't need to make an appointment to visit me during office hours; just come by! If you can't make it to my scheduled office hours, you are always welcome to email me, and we can set up another time to meet.

  I'll announce the times of my office hours soon, after I've had a chance to look over your schedules.
- *CA problem sessions*: Course assistant(s) will hold weekly problem sessions, and you may attend any CA's problem session. Times will be posted on the course website soon.
- *Math Night*: Math Night will be held every Monday night (starting Sep. 10), 8-10PM at the Leverett house dining hall. It is hoped that you will find students from Math 141a (or other math classes) there to work on problem sets or get help with them. "Extra special" food is promised. The website for Math Night is:
  `http://math.harvard.edu/undergrad/mathnight.html`.

Other policies.

Contacting me. Feel free to talk to me anytime. I will often be in my office, and you are welcome to drop by, but I might tell you I am busy if you come outside regular office hours. The best way to otherwise contact me is via email, as it provides me with a written record of our conversation.

Attendance. I strongly encourage you not only to attend lectures, but also to actively participate in them: stop me if anything is unclear and feel free to share your thoughts about the material (what do you find easy? What do you find hard?).

I expect your full attention during lectures: no loud conversation, use of cell phones, or other activities unrelated to class.

Grading issues. If you have any questions or complaints concerning the way an assignment has been graded that cannot be resolved with the CA, please come talk to me. You should first look at the official solution and make sure you understand it.

Academic integrity. Any acts of academic dishonesty, such as cheating, plagiarism, etc. will be dealt with according to University Policy. Examples of violation include searching the web (or inside a textbook) for solutions, copying part of another student's assignment or showing your assignment to another student. Please speak to me if you have any questions about this.