MATH 145A - SET THEORY I, FALL 2019
ASSIGNMENT 1

Due Tuesday, September 10 at the beginning of class (please submit your assignment as a PDF on Canvas). Make sure to include your full name *and the list of your collaborators* (if any) with your assignment. You may discuss problems with others, but you may *not* keep a written record of your discussions. Please refer to the syllabus for details.

As a general rule, imagine that you are writing your solution to convince somebody else in the class who is very skeptical about the particular statement. In particular, it should be completely understandable to another student: always justify your reasoning in plain English. It is not sufficient to simply state a number or formula without providing the steps and reasoning that you used to produce the answer.

(1) (Extra credit: 15%)
   (a) Please fill in the survey at: `http://math.harvard.edu/~sebv/155r-fall-2019/questionnaire.odt`. Submit it separately on Canvas.
   (b) I like to know my students as human beings, so I would like to have a short one on one 5-10 min chat with you during the first few weeks of the semester, just so that I can know your face, name, and a little bit about your background. Don't be afraid, we're not going to talk math (unless you really want to!). You don't need to prepare anything for the meeting.
   Please send me a short email at `sebv@math.harvard.edu` with subject "145a short meeting" and ask e.g. "is 1pm next Monday okay?". I will either reply yes or propose another time. The meeting will take place in my office, SC 321H.

(2) Let E be an equivalence relation on a set A. Prove that its set A/E of equivalence classes is a partition on A. Conversely, if P is a partition of the set A, prove that there is an equivalence relation E_P on A so that $A/E_P = P$.

(3) (a) Prove that if A and B are sets, then $A \times B$ is a set.
   (b) Prove that if A and B are sets, then the class ${}^{A}B$ of all functions from A to B is a set.

(4) (a) Prove that the composition of two injections is an injection, and that the composition of two surjections is a surjection.
   (b) Prove that a function $f : B \to C$ is an injection if and only if whenever $g_1, g_2 : A \to B$ are functions with $f \circ g_1 = f \circ g_2$ then $g_1 = g_2$. State a similar characterization for surjections (no need to prove it).
   (c) Prove that any function f can be written as $f = g \circ h$ for some injection g and some surjection h. Prove also that $f = g' \circ h'$ for some surjection g' and some injection h'.

Date: September 2, 2019.

(5) (a) Prove that if $n \in \mathbb{N}$, then $n \subseteq \mathbb{N}$.
(b) Prove that if $n, m \in \mathbb{N}$ and $n \in m$, then $n \subseteq m$.
(c) Prove that if $n \in \mathbb{N}$, then $n \notin n$.
(d) Prove that for n, m in $\mathbb{N}$, $n \in m$ if and only if $n \subsetneq m$. Deduce that $\in$ is an irreflexive, antisymmetric, transitive relation on $\mathbb{N}$.
(e) Recall that addition on $\mathbb{N}$ is defined using the following properties, for any $n, m \in \mathbb{N}$:
- $n + 0 = n$.
- $n + (\mathrm{S}\, m) = \mathrm{S}(n + m)$.

Using only these properties, prove that addition is commutative. That is, for $a, b \in \mathbb{N}$, $a + b = b + a$. *Hint for all parts: induction.*