MATH 145A - SET THEORY I
INFORMATION ON THE FINAL PROJECT

What? The final project is an opportunity for you to follow your curiosity and connect the mathematics we have been learning with your broader interests and coursework. You could either investigate applications and connections of set theory to other fields, or explore a topic of set theory more deeply. The project is individual, but it is okay to take the same topic as others and discuss it in group. However the final writeup should be your own and you cannot keep records of your discussions: the same rules as for the assignments apply.
The goal is to learn about a topic and write an 8-12 page exposition. The topic should be new to you and should not have been covered in class. You do *not* need to (and are *not at all* expected to) come up with an original research idea: this takes years! I just expect you to read about some cool math, understand it, and write your own exposition.
Your report should include a clear introduction that motivates your topic and sets the stage for the rest of the report. I expect to see serious mathematical content, including definitions, theorems, and proofs. The report should also *cite* all the references that you used.
You should typeset your project (preferably with latex, but this is not required). You should also submit a paragraph listing your collaborators and their contributions to the project.
The rest of this handout gives the key deadlines, the grading rubric and presents some potential project ideas. If there's some other topic you're interested in writing about, you are more than welcome to. I highly recommend talking with me about project ideas.
When? Below are the key dates:

- Tuesday, October 29: *Project proposal due* (this will part of assignment 8). Your proposal should include your topic, and a rough outline of your project (about a page). This outline should address the following questions:
  - What are your main motivating questions or problems?
  - What sources do you plan to read?
  - What theorems do you plan to write a proof of?
- Tuesday, November 19: *Project draft due.*

Date: October 7, 2019.

- Tuesday, November 26: *Peer review due.* You will have to read somebody else's draft, give feedback, and suggest improvements (about a page is enough).
- Thursday, December 5: *Final project due.*

How will this be graded?
The final project will be graded out of 100 points (the breakdown is below), and will count for 15% of your final grade.

- Draft is turned in on time and is reasonably complete (10 points).
- Peer review contains valuable feedback (5 points).
- Draft feedback was incorporated (5 points)
- Motivation and Introduction (10 points):
  - Discussion of why the topics are interesting.
  - Clear connections with course topics.
  - Clear motivating questions.
  - How does this fit in the bigger picture?
- Communication and Writing Style (15 points)
  - Ideas are clearly communicated.
  - Correct grammar and spelling, good mathematical writing (for example, using precise language)
  - Is the paper readable? Are there clear transitions between topics?
  - Material is explained at the level of a student in the class.
- Definitions (10 points):
  - Clear, correct, and precise mathematical definitions.
- Examples (10 points):
  - Examples are correct, complete and well-chosen.
  - There are sufficient examples to help the reader understand key definitions, theorems, and proofs.
- Mathematics (20 points):
  - Clear statements of theorems/propositions/lemmas/corollaries/etc.
  - Correct and consistent use of notation.
  - Complicated proofs are broken up into smaller pieces.
  - Correct and concise mathematical proofs.
  - Content and techniques are at the level a student in the class would understand.
  - Arguments are complete, and details are provided.
- Bibliography (5 points):
  - Citations are correct and complete.
  - Citations come from reputable sources.
- Overall cohesiveness (10 points):
  - The topics are in a logical order.

– The topics fit together into a coherent story or theme.
– Collaborators are acknowledged.

Some ideas of projects

Some of the topics below are covered in the additional references listed on the course webpage. Many are not: Wikipedia or a search engine are your best tools to quickly find an overview and links to further references. I'm also happy to suggest references. As said above, the projects below are only suggestions, but if you would like to do a completely different topic I highly recommend talking to me.

- The Banach-Tarski paradox, and other strange consequences of the axiom of choice (c.f. an article of Hardin and Taylor titled "a peculiar connection between the axiom of choice and predicting the future").
- Equivalents of the axiom of choice (c.f. Jech's book: the axiom of choice).
- Forcing and the independence of the continuum hypothesis (a good reference to start is Timothy Chow's article).
- Other foundations for mathematics: category theory (Tom Leinster's *rethinking set theory*) or type theory (the book *Homotopy type theory*).
- Invent your own set theory with classes and superclasses ("conglomerates"), and write out the axiom and some of the basic lemmas in details.
- Partition relations, including the Erdos-Rado theorem.
- More cardinal arithmetic: investigate some of the bounds by Galvin-Hajnal, write about Shelah's pcf theory and the famous consequence that if $\aleph_\omega$ is strong limit, $2^{\aleph_\omega} < \aleph_{\omega_4}$.
- More descriptive set theory. See the books by Kechris and by Moschowakis. Possible theorems: Π^1_1-uniformization, investigation of more games (the Banach-Mazur game, the covering game), consequences of the axiom of determinacy (for example: the club filter is an ultrafilter – $\aleph_1$ is a large cardinal).
- The Diamond principle and Suslin trees. Diamond is a combinatorial principle that holds in the constructible universe. Suslin trees are closely related to *Suslin's problem*: is there an ordering satisfying the following properties that is not isomorphic to the real line? Dense, complete, without endpoints, every collection of disjoint open sets is countable.
- Martin's axiom.

- Whitehead's problem: a problem in algebra now known to be independent of the standard axioms of set theory.
- Does almost free imply free? Shelah's singular compactness theorem. See Wilfried Hodges, *In singular cardinality, locally free algebras are free*.
- Investigate a large cardinal principle of your choice! Vopenka's principle may be an interesting one to look at for example.
- Prove Borel determinacy.
- The anti-foundation axiom.
- Cardinality in categories (disclaimer: this one of the research areas I specialize in): the notion of an accessible category and the presentability rank of an object. This is a way to relativize cardinality to specific classes of algebraic objects. For example instead of "finite" one obtains "finite dimensional" when specializing to vector spaces. See the paper "Accessible categories, set theory, and model theory: an invitation" on my webpage.