# MATH 145A - SET THEORY I, FALL 2019

TR 12-1h15PM, SC 310

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**Course assistants:** TBD

**Course website:** http://math.harvard.edu/~sebv/145a-fall-2019/

**Course overview**. The course will be an introduction to set theory. Set theory studies the actual infinite, and questions such as when an infinity is bigger than another, how to "catch one's tail" by repeatedly performing a sequence of steps infinitely-many times, how to perform recursive constructions that last a large infinity a steps ("transfinite induction"), and how to measure the complexity of infinite objects (such as sets of reals). A foundational question is the continuum hypothesis: any uncountable set of real numbers should be in bijection with the full set of real numbers. In fact, fundamental to set theory is a deep theory of "infinite numbers" (ordinals and cardinals) which extend the finite natural numbers. These objects were introduced by Georg Cantor, who (it is believed) later went mad trying to prove the continuum hypothesis.

Initially, set theory was hailed as a foundation for all of mathematics: any mathematical object can, in principle, be encoded as a set. We will sketch that approach, but will not emphasize it, since there are others appropriate foundations for mathematics. On the other hand, we will try to emphasize ideas from set theory that are mathematically beautiful in their own right, as well as tools that are more likely to be useful for "regular" mathematics (e.g. analysis or topology). For example, we will use transfinite induction to analyze the structure of closed sets of reals (and deduce that closed sets satisfy the continuum hypothesis). We will also talk about infinite games and their determinacy.

Another highlight of the class will be the proof of consistency of the continuum hypothesis (by Kurt Gödel): we will build a universe of sets, called the *constructible universe* in which the continuum hypothesis holds. Many years after Gödel, Paul Cohen earned a Fields medal

by showing that the continuum hypothesis can fail in other universes of sets (exploring Cohen's proof could be your final project topic for the class). As a result, it is (provably) impossible to prove or disprove the continuum hypothesis. Several other questions about cardinal arithmetic remain open to this day, but nevertheless there is a beautiful *infinite combinatorics* involving them, which we will also explore (we will for example prove *Silver's theorem* about the cardinal arithmetic of so-called singular cardinals).

**Course texts**. We will not use a primary textbook, but online notes will regularly be posted on the course website. Other *optional* references can be found on the course website (more may be added as class proceeds).

**Prerequisites**. The most important prerequisite is familiarity with proofs. For example you should know how to do (and write) a proof by contradiction, cases, induction, etc. You should know what a relation is. A previous course at the level of Mathematics 23a, 25a, 55a, 101, 102, 112, or CS-20, would be enough.

**Assessment**. Your grade for the course will be determined by scores on homework assignments, two exams, and a final project as follows:

- There will be two exams: a midterm (in class), and a final (take home). They will collectively count for *50%* of your final grade. Your lowest score will count for *20%* and the other will count for *30%*.
- There will be a *final project*, which will count for *15%* of your final grade.
- There will tentatively be 12 *homework assignments*. Cumulatively, they will count for *35%* of your final grade. Your lowest homework score will be dropped. See below for more details on homework assignments.

The *highest* grade cutoffs will be as follows: 90%: A, 80%: B, 70%: C, 60%: D. These cutoffs *might* be lowered, but will not be raised. This means, for example, that you will be guaranteed an A if you score 90% or more, regardless how the rest of the class performs. If you are borderline (for example you score 89% and the threshold for A ends up staying at 90%), then you *may* be bumped up to $B^+$ or $A^-$ depending on factors such as class participation, especially insightful project or assignment solutions, etc.

If you qualify for special accommodation (such as extra time) for the tests, or if you already know you will not be able to take one of the

tests at the planned time (e.g. because of a religious observation or a university event), please let me know as soon as possible.

**Exams**. There will be two exams that will take place during the semester. The (tentative) dates are below:

- The *midterm* will take place in class on Thursday, October 17 from 12 to 1h15pm.
- The *take-home final* will be out on Tuesday, December 3 at 1h15pm and due 48 hours later (Thursday, December 5, 1h15pm).

**Project**. For your *final project*, you will explore a set theory topic of your choice that was not covered in class (a list of possible topics will be available, but you will also be able to select something else). You will have to produce a written introduction to this topic. This will give you the occasion to read about something that interests you and "make it your own" by explaining it in your own words. This project will have to be written individually, but collaboration will be encouraged. Your final writeup will be peer-reviewed by other students. Further details will be provided during the last third of the semester.

**Homework assignments**. They will be announced in class and posted on the course website. Solutions will typically be posted the day after an assignment is due. Assignments will usually be due at the *beginning* of class. Only partial credit will be given if an assignment is turned in late, and no credit if it is turned in after solutions have been distributed. Assignments have to be submitted *online*, via the course's Canvas site (good quality scans are acceptable, but using latex is encouraged).

Assignments are one of the key elements of this course (topics that have not been seen in class may be introduced there). You should make every effort to write down your thoughts clearly and precisely. Your writeup should contain little to no extraneous material (no scrap work). I also encourage you to *be intellectually honest*: it is better to say that you are not exactly sure how to solve a problem / justify a particular step and write your thoughts than to write three pages of obscure equations and hope the grader will trust your solution to be correct.

On the first page of your assignment please include:

- Your full name.
- The *list of other students with whom you collaborated* (if any).

As long as you list your collaborators, collaboration is allowed and encouraged. You may discuss ideas on, and even possible solutions of, specific problems. *However*, you may *not* maintain a record (written,

audio, photographic, etc.) of the discussion. This means that *you are required to write up solutions entirely on your own* and that you cannot show the assignment you are submitting to other students. For example, if you discuss a problem with others using a blackboard, you must erase the board once the discussion is over and write up the solution on your own.

**Reading assignments**. The reading assignments that are relevant for the lectures of any given week will be posted to the course website at the end of the previous week. You will benefit from looking at the reading (if only for a few minutes) before coming to class.

**Resources for help**. Many resources outside of class are offered, and I highly encourage you to take advantage of them:

- *Office hours (in my office, SC 321H)*: Office hours are times when I will be sitting in my office, just waiting for you to come ask me anything related to this class. You don't need to make an appointment to visit me during office hours; just come by! If you can't make it to my scheduled office hours, you are always welcome to email me, and we can set up another time to meet.

  I'll announce the times of my office hours soon, after I've had a chance to look over your schedules.

  CAs will also hold office hours (to be annouced).
- *CA problem sessions*: Course assistant(s) will hold weekly problem sessions, and you may attend any CA's problem session. These are time where you can get more practice on the course material and get another opportunity to ask questions and clarify anything unclear. Times will be posted on the course website soon.
- *Math Night*: Math Night will be held every Monday night, 8-10PM at the Leverett house dining hall. It is hoped that you will find students from Math 145a (or other math classes) there to work on problem sets or get help with them. "Extra special" food is promised. The website for Math Night is: `http://math.harvard.edu/undergrad/mathnight.html`.

**Other policies.**

**Contacting me**. Feel free to talk to me anytime. I will often be in my office, and you are welcome to drop by, but I might tell you I am busy if you come outside regular office hours. The best way to otherwise contact me is via email, as it provides me with a written record of our conversation.

**Attendance**. I strongly encourage you not only to attend lectures, but also to actively participate in them: stop me if anything is unclear and feel free to share your thoughts about the material (what do you find easy? What do you find hard?).

I expect your full attention during lectures: no loud conversation, use of cell phones, or other activities unrelated to class.

**Grading issues**. If you have any questions or complaints concerning the way an assignment has been graded that cannot be resolved with the CA, please come talk to me. You should first look at the official solution and make sure you understand it.

**Academic integrity**. Any acts of academic dishonesty, such as cheating, plagiarism, etc. will be dealt with according to University Policy. Examples of violation include searching the web (or inside a textbook) for solutions, copying part of another student's assignment or showing your assignment to another student. Please speak to me if you have any questions about this.

**Wellness**. Take care of yourself. Do your best to maintain a healthy lifestyle this semester by eating well, exercising, avoiding drugs and alcohol, getting enough sleep and taking some time to relax. This will help you achieve your goals and cope with stress.

All of us benefit from support during times of struggle. You are not alone. There are many helpful resources available on campus and an important part of the college experience is learning how to ask for help. Asking for support sooner rather than later is often helpful.

If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, I strongly encourage you to seek support. Counseling and Mental Health Services (CAMHS) is here to help: call 617-495-2042 or visit their website at `https://camhs.huhs.harvard.edu/find-help-now` Consider reaching out to a friend, faculty or family member you trust for help getting connected to the support that can help.

If you have questions about this or your coursework, please let me know.