MATH 155R - COMBINATORICS
INFORMATION ON THE LAST EXAM

When? The last exam will be distributed right after class on Tuesday, December 3 at 11h45am. It will also be posted on the course webpage at the beginning of the afternoon on December 3. You can take it home and will have 48 hours to solve the problems. The exam will be due back on Thursday, December 5, before 11h45am. Please send it as a PDF to me by email (I will reply with a confirmation): `sebv@math.harvard.edu`

If you cannot take the exam at that time (for example because of a religious holiday or a university event), please let me know as soon as possible.

What is covered? The last exam will cover everything discussed in class up to and including the lecture on Thursday, November 21. This also includes the topics covered in assignments 1-12. Emphasis will be on the topics seen in class *after* the midterm. For your convenience, the following lists the topics discussed in class that were not covered in the midterm, with corresponding chapters and sections from Matoušek-Nešetřil.

- *Graph drawings (chapter 6)*: planar graphs and Euler's formula (6.3), the chromatic number of a graph, the five color theorem (6.4). We did not prove 6.3.3(ii) fully, and did not discuss 6.3.4, or the second proof of 6.4.5.
- *Double counting (chapter 7)*: Sperner's lemma, the Brouwer fixed point theorem, the game of Hex cannot end in a draw (all in 7.1). Sperner's theorem on independent systems (7.2). For 7.2, we discussed the first and third proof, not the second.
- *Finite projective planes (chapter 9)*: Definition and basic properties, the Fano plane, the incidence graph, duality (9.1). Existence (9.2), connection with Latin squares (9.3), The Fano plane is not 2-colorable, the incidence graph of a projective plane gives a good lower bound for the number of edges of a $K_{2,2}$-free graphs (9.4).
- *The probabilistic method (chapter 10)*: Four rounds of dovetail shuffling are not enough, results on 2-colorings of set systems (10.1 – the application to boolean functions was not discussed). Definition and examples of finite probability spaces, the game described on p. 294 (called the "job interview puzzle" in class), random graphs, independence of events, existence of transitive tournaments (all in 10.2). Definition of random variables and their expectations, linearity of expectation, examples 10.3.1 and 10.3.2 (the number of heads in a coin toss and the number of surviving rabbits – the other examples

in 10.3 were not discussed), Three applications: existence of large bipartite subgraphs, Turán's theorem, and the number of intersections of level at most k (10.4). The last application (number of comparisons of quicksort) was not discussed. However we did discuss one final application: the Erdős-Ko-Rado theorem (notes on the course website, under "other course documents").

- *Ramsey's theorem (chapter 11)*: The Ramsey numbers, upper bound (11.2) and lower bound using the probabilistic method (11.3).

What can I use during the exam? You may use your personal course notes, the course textbook (Jiří Matoušek and Jaroslav Nešetřil, *Invitation to discrete mathematics*, 2nd edition, Oxford University Press, 2008), the notes under "other course documents" (including the Erdős-Ko-Rado theorem) from the course website, the assignments, sample exams, midterm, and their solutions posted on Canvas. You may *not* use any other documents. In particular, you are *not* allowed to search the web for solutions, go on Wikipedia, or consult any other textbooks.

What else can I use? Only your brain, blank paper, and a pen (or maybe a computer and software to type). You may *not* use a calculator (it would be completely useless anyway) and you may *not* collaborate with anybody. You are *not* even allowed to show the exam to anybody (even if you have no plans to collaborate with them) before Dec. 5, 11h45am.

What will the exam look like? The exam will have six problems, you will be asked to solve four of them. There will also be one extra credit problem. I will try to make the problems slightly easier than in your homework. Your answer will be expected to have the same level of details as in the homework.

What can I do to prepare for the exam? Make sure you understand all of the homework and the course material. A sample exam is available on the course website. Try to solve it! You can also try to do the problems at the end of each section in the textbook (the first few problems are usually easier). You can find solution and/or hints to some of those at the end of the book.