

**MATH 155R - COMBINATORICS, FALL 2019**  
**GEORGE'S TWO MAGICIANS PROBLEM**

*The following problem was shared by George. No formal credit will be given for a solution, and it is not needed for the class. Just do it for fun!*

Magicians Abra and Cadabra are planning to perform the following trick. At the start, Abra locks Cadabra inside a room where she can see and hear nothing. Then Abra invites a member of the audience to place a black or a white counter on each square of an  $n \times n$  chessboard, and then choose a square  $C$  of the chessboard. Next, Abra chooses exactly one square  $D$  of the chessboard (which may be the same as  $C$ ), and replaces the counter on  $D$  with a counter of the other colour (changing it from white to black, or black to white).

The room in which Cadabra was locked is now opened, and Cadabra, just looking at the chessboard, guesses which square is  $C$ . For what values of  $n$  do Abra and Cadabra always have a winning strategy, regardless of the audience member's choices?