MATH 155R - COMBINATORICS
INFORMATION ON THE FINAL PROJECT

What? The final project is an opportunity for you to follow your curiosity and connect the mathematics we have been learning with your broader interests and coursework. You could either investigate applications and connections of combinatorics to other fields, or explore a topic of combinatorics more deeply. The project is individual, but it is okay to take the same topic as others and discuss it in group. However the final writeup should be your own and you cannot keep records of your discussions: the same rules as for the assignments apply.
The goal is to learn about a topic and write an 8-12 page exposition. The topic should be new to you and should not have been covered in class. It can either be from the book or from some other source. You do *not* need to (and are *not at all* expected to) come up with an original research idea: this takes years! We just expect you to read about some cool math, understand it, and write your own exposition.
Your report should include a clear introduction that motivates your topic and sets the stage for the rest of the report. We expect to see serious mathematical content, including definitions, theorems, and proofs. The report should also *cite* all the references that you used.
You should typeset your project (preferably with latex, but this is not required). You should also submit a paragraph listing your collaborators and their contributions to the project.
The rest of this handout gives the key deadlines, the grading rubric and presents some potential project ideas. If there's some other topic you're interested in writing about, you are more than welcome to. I highly recommend talking with me or George (and ideally both of us!) about project ideas.
When? Below are the key dates:

- Tuesday, October 29: *Project proposal due* (this will part of assignment 8). Your proposal should include your topic, and a rough outline of your project (about a page). This outline should address the following questions:
  - What are your main motivating questions or problems?
  - What sources do you plan to read?
  - What theorems do you plan to write a proof of?

Date: October 8, 2019.

- Tuesday, November 19: *Project draft due.*
- Tuesday, November 26: *Peer review due.* You will have to read somebody else's draft, give feedback, and suggest improvements (about a page is enough).
- Thursday, December 5: *Final project due.*

How will this be graded?
The final project will be graded out of 100 points (the breakdown is below), and will count for 15% of your final grade.

- Draft is turned in on time and is reasonably complete (10 points).
- Peer review contains valuable feedback (5 points).
- Draft feedback was incorporated (5 points)
- Motivation and Introduction (10 points):
  - Discussion of why the topics are interesting.
  - Clear connections with course topics.
  - Clear motivating questions.
  - How does this fit in the bigger picture?
- Communication and Writing Style (15 points)
  - Ideas are clearly communicated.
  - Correct grammar and spelling, good mathematical writing (for example, using precise language)
  - Is the paper readable? Are there clear transitions between topics?
  - Material is explained at the level of a student in the class.
- Definitions (10 points):
  - Clear, correct, and precise mathematical definitions.
- Examples (10 points):
  - Examples are correct, complete and well-chosen.
  - There are sufficient examples to help the reader understand key definitions, theorems, and proofs.
- Mathematics (20 points):
  - Clear statements of theorems/propositions/lemmas/corollaries/etc.
  - Correct and consistent use of notation.
  - Complicated proofs are broken up into smaller pieces.
  - Correct and concise mathematical proofs.
  - Content and techniques are at the level a student in the class would understand.
  - Arguments are complete, and details are provided.
- Bibliography (5 points):
  - Citations are correct and complete.
  - Citations come from reputable sources.
- Overall cohesiveness (10 points):

– The topics are in a logical order.
– The topics fit together into a coherent story or theme.
– Collaborators are acknowledged.

Some ideas of projects

Some of the topics below are covered in the course textbook. Many are not: Wikipedia or a search engine are your best tools to quickly find an overview and links to further references. Feel free to ask me if you need suggestions for references. As said above, these are only suggestions, but if you would like to do a completely different topic I highly recommend talking to me and/or George first.

- The art gallery problem: how many people do you need to guard an art gallery?
- The coloring number of the plane: how many colors do you need to color the plane in such a way that no two points at distance exactly one have the same color? (A biologist at MIT recently made a breakthrough on this!)
- Sperner's lemma: you could investigate more applications such as the cake cutting problem and fair rent division, or you could look at generalizations (such as Sperner's lemma for polytopes). Reference on this (and several other lemmas): *The discrete yet ubiquitous theorems of Carathéodory, Helly, Sperner, Tucker, and Tverberg*. Bulletin of the AMS
- Prove the equivalence of the determinacy of Hex with Brouwer's fixed point theorem (without going through Sperner's lemma).
- Investigate Chapter 8 of the book, and the many proofs explaining why the number of spanning trees on n vertices is n^{n-2}.
- More Ramsey theory: Van der Warden theorem or the Hales Jewett theorem. You could also try to understand how to compute the Ramsey numbers in more details (why is $R(4) = 18$? Why is it so hard to compute $R(5)$? What about asymmetric Ramsey numbers such as $R(3, 9)$?).
- Matroid theory. You could for example investigate the definition of a *greedoid*: a generalized setup for greedy algorithms.
- Möbius inversion and incidence algebras: generalize the inclusion-exclusion principle by thinking in terms of posets. See *On the Applications of Möbius Inversion in Combinatorial Analysis*. American Mathematical Monthly.
- More on the probabilistic method, including the Lovász local lemma. See the book by Alon and Spencer on the probabilistic method.

- Information theory: study codes for digital communication. The most famous is the Reed-Solomon code.
- The game SET and the cap set conjecture. See *New applications of the polynomial method: The cap set conjecture and beyond*. Bulletin of the AMS.
- Kuratowski's theorem: a graph is planar if and only if it does not have a subgraph isomorphic to a subdivision of K_5 or $K_{3,3}$.
- More books that may have some interesting topics: the other references from the course website. *Proofs from the book*, *Graph theory* by Diestel.