# EXERCISE 2.3.21: MULTIPLYING BY THE CONJUGATE

SEBASTIEN VASEY

## Exercise

Compute the following limit (if it exists):

$$\lim_{h \to 0} \frac{\sqrt{9+h}-3}{h}$$

## Solution

Plugging in $h = 0$ leads an indeterminate expression of the form $\frac{0}{0}$, so some simplification is needed. We use a simple trick worth remembering: we multiply by one, writing one as something which is convenient to simplify the expression.

$$\begin{aligned}
\lim_{h \to 0} \frac{\sqrt{9+h}-3}{h} &= \lim_{h \to 0} \frac{\sqrt{9+h}-3}{h} \cdot 1 \\
&= \lim_{h \to 0} \frac{\sqrt{9+h}-3}{h} \cdot \frac{\sqrt{9+h}+3}{\sqrt{9+h}+3} \\
&= \lim_{h \to 0} \frac{9+h-9}{h(\sqrt{9+h}+3)} \\
&= \lim_{h \to 0} \frac{h}{h(\sqrt{9+h}+3)} \\
&= \lim_{h \to 0} \frac{1}{\sqrt{9+h}+3} \\
&= \frac{1}{\sqrt{9+0}+3} \\
&= \frac{1}{6}
\end{aligned}$$

$\sqrt{9+h}+3$ is called the *conjugate* of $\sqrt{9+h}-3$. Multiplying by the conjugate is sometimes used to get rid of square roots, as seen in this example.