HANDOUT: SEP. 03, 2013 RECITATION

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Exercise 2.3.39

(This was done with the 11h30 recitation, but not with the 9h30 recitation).
Prove that

$$\lim_{x \to 0} x^4 \cos(\frac{2}{x}) = 0$$

Solution

Note that $-1 \leq \cos(\frac{2}{x}) \leq 1$, so by the Squeeze theorem,

$$\lim_{x \to 0} x^4(-1) \leq \lim_{x \to 0} x^4 \cos(\frac{2}{x}) \leq \lim_{x \to 0} x^4(1)$$

So

$$0 \leq \lim_{x \to 0} x^4 \cos(\frac{2}{x}) \leq 0$$

and the result follows. Note that we cannot use the product rule for limits, as $\lim_{x \to 0} \cos(\frac{2}{x})$ does not exist: the function oscillates very quickly between -1 and 1 near 0.

Exercise 2.2.35

Compute

$$\lim_{x \to 2\pi^-} x \csc(x)$$

Solution

Recall that $\csc(x) = \frac{1}{\sin(x)}$, then the limit is of the form $\frac{2\pi}{0}$, and $\sin(x)$ is negative when $x < 2\pi$, but x is close to 2π (draw a picture). Thus the limit is $-\infty$.

Exercise 2.2.37

Compute

$$\lim_{x \to 2^+} \frac{x^2 - 2x - 8}{x^2 - 5x + 6}$$

Solution

This is a limit of the form $\frac{-8}{0}$, so we want to know the sign of $x^2 - 5x + 6$ as $x > 2$, x close to 2. Simply e.g. plotting this function, one sees that it is less than zero between 2 and 3, so the limit is $+\infty$.