MATH 269X - MODEL THEORY FOR ABSTRACT ELEMENTARY CLASSES, SPRING 2018

MWF 11AM-12PM, SC 310

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Course website: http://math.harvard.edu/~sebv/aec-spring-2018/

Course overview. Abstract elementary classes are a kind of concrete categories with all directed colimits whose morphisms are monomorphisms. They generalize the model theory of classes axiomatized by a first-order theory or certain infinitary logics, and encompass many algebraic classes of objects such as valued fields or locally finite groups. We will give a presentation of both basic model-theoretic results (the presentation theorem, model-homogeneous is saturated, closure under infinitary elementary equivalence) and generalizations of stability theory to this setup, with a focus on free (nonforking) amalgamation (generalizing linear independence of vector spaces and algebraic independence in fields). We will also discuss the connections with combinatorial set theory and large cardinals.

Course text. No textbook is required for this class. Instead, lecture notes will be regularly posted on the course website and they will heavily rely on the literature. Below are some optional references.

There are currently two textbooks on abstract elementary classes. They emphasize different aspects:

- Saharon Shelah, *Classification theory for abstract elementary classes*. Studies in Logic: Mathematical logic and foundations, vol. 18 & 20, College Publications. 2009.
- John T. Baldwin, *Categoricity*. University Lecture Series, vol. 50, American Mathematical Society, 2009.

Shelah's books and papers can be hard to read at places but they are impossible to avoid. The introduction of his book is an accessible and quite easy read (Paper E53 on Shelah's list).

Many of the chapters of Shelah's books are available online (see the course website for links). Baldwin's book is also available from his webpage.

The following survey papers may also be helpful:

- Rami Grossberg, *Classification theory for abstract elementary classes*. Logic and Algebra, ed. Yi Zhang, Contemporary Mathematics, Vol 302, AMS, (2002), 165 204.
- John T. Baldwin, *Abstract Elementary Classes: Some Answers, More Questions*. Logic Colloquium 2004, Turino. Available online: `http://www.math.uic.edu/~jbaldwin/pub/turino2.pdf`.
- Will Boney and Sebastien Vasey, *A survey on tame abstract elementary classes*. Beyond First Order Model Theory (José Iovino ed.), CRC Press (2017), 353–427.

Prerequisites. I will assume acquaintance with basic logic and set theory. Some familiarity with elementary model theory (e.g. Chang and Keisler's book) will be helpful.

Assessment. If you are taking this course for a grade, please let me know and I will give you a small reading project to do at the end of the semester.