CONCEPTS OF MATHEMATICS, SUMMER 1 2014
ASSIGNMENT 2

Due Wednesday, May 28 at the beginning of class. Make sure to include your name, Andrew ID, *and the list of your collaborators* (if any) with your assignment. You may discuss problems with others, but you may *not* keep a written record of your discussions. Please refer to the syllabus for details.

Problem 1 (20 points)

(1) Prove Theorem 3.20 from the notes: For all real numbers x and y:
   (a) $(xy)^2 = x^2y^2$.
   (b) If x and y are non-negative, $\sqrt{xy} = \sqrt{x}\sqrt{y}$.

(2) Prove Theorem 3.22 from the notes: For all real numbers x and y:
   (a) $x^2 = |x|^2$.
   (b) $|x| = \sqrt{x^2}$.
   (c) $x \leq |x|$.
   (d) $|xy| = |x||y|$.

Problem 2 (20 points)

Say whether each of the following statement is true or false. If it is true, prove it. If it is false, give a counterexample.

(1) For all non-negative real numbers x and y, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.
(2) For all non-negative real numbers x and y, $\sqrt{x+y} \leq \sqrt{x} + \sqrt{y}$.
(3) For all non-negative real numbers x and y, $\sqrt{x+y} \geq \sqrt{x} + \sqrt{y}$.
(4) For all non-negative real numbers x and y, $\sqrt{2(x+y)} \geq \sqrt{x} + \sqrt{y}$.

Problem 3 (20 points)

Assume p, q, r are propositions.

(1) Prove that $p \to q \equiv \neg q \to \neg p$. The statement $\neg q \to \neg p$ is called the *contrapositive* of $p \to q$.
(2) Prove the following distributive law: $r \wedge (p \vee q) \equiv (r \wedge p) \vee (r \wedge q)$.

Date: May 23, 2014.

(3) Find a proposition involving p, q that is true precisely when *exactly* one of p and q are true.

(4) Write a proposition logically equivalent to the negation of $p \to q$ that does not contain "outer" negations, i.e. your proposition can only include brackets, $\wedge$, $\vee$, $\to$, $\leftrightarrow$, T, F, p, q, $\neg p$, or $\neg q$ (so $\neg(p \to q)$ does not have the right form).

Problem 4 (40 points)

Assume p and q are propositions. Define the logical operator NAND by the following truth table:

p	q	p NAND q
F	F	T
F	T	T
T	F	T
T	T	F

You should convince yourself that p NAND q is logically equivalent to $\neg(p \wedge q)$.

Show that $F, T, \neg p, p \vee q$, $p \wedge q$, $p \to q$, and $p \leftrightarrow q$ can all be expressed using only NAND, p, and q.