CONCEPTS OF MATHEMATICS, SUMMER 1 2014
ASSIGNMENT 3

Due Friday, May 30 at the beginning of class. Make sure to include your name, Andrew ID, *and the list of your collaborators* (if any) with your assignment. You may discuss problems with others, but you may *not* keep a written record of your discussions. Please refer to the syllabus for details.

Problem 1 (20 points)

Assume x and y are real numbers, and assume we know that y is greater than or equal to every real number less than x.

(1) One can see this second assumption as a propositional function with variables x and y and domain of discourse the real numbers. Write this propositional function in symbols.
(2) Show that $x \leq y$.

Problem 2 (20 points)

Assume n and m are integers. Complete the proof of Theorem 5.5 in the notes, namely show:

(1) If n is even, then nm is even.
(2) If n and m are odd, then nm is odd.

Problem 3 (20 points)

For each of the statement below, say whether it is a proposition or only a propositional function, and write it in symbols using the integers as domain of discourse.

(1) x is an even integer.
(2) x is an odd integer.
(3) Any integer is either even or odd.
(4) Any even integer is not odd.

Date: May 26, 2014.

Problem 4 (20 points)

Write the negation of the following propositions in symbols and without outer negations, namely "$\neg\exists$", "$\neg\forall$", and "$\neg($" cannot appear in your final answer.

Also determine the truth value of those propositions (and justify). Use the real numbers as domain of discourse.

(1) $\exists x(0 = 1 \to (x = 3 \land x = 2))$
(2) $\forall x\,(x \geq 0 \to (x \neq 2 \to (0 = 1 \lor x \neq 3)))$
(3) $\forall x \exists y \exists z(z \geq 0 \land (y = -1 \lor y = 1) \land x = yz)$
(4) $\exists y \exists z \forall x(z \geq 0 \land (y = -1 \lor y = 1) \land x = yz)$

Problem 5 (20 points)

CMU students celebrate the end of the semester at a party. We assume any two students either know or do not know each other, and that knowledge is a symmetric and transitive relation: if x knows y, then y also knows x, and if x knows y and y knows z, then x knows z.

(1) Write the above assumptions in symbols, as a proposition over the domain of discourse of students attending the party. Use xKy to say that student x knows student y.
(2) Show that if at least five students come to the party, there will exist three (distinct) students such that either the three all know each other, or the three all do not know each other. Is this still true if we replace five by four?