CONCEPTS OF MATHEMATICS, SUMMER 1 2014
SAMPLE MIDTERM EXAM

First name: ______________________________

Last name: ______________________________

Andrew ID: ______________________________

You have one hour and twenty minutes to complete the exam. The exam should contain five problems and one extra credit question.

You may *not* use any notes or other documents, and you may *not* use any electronics such as a calculator or a mobile phone. You may use your brain and a pen.

Quote of the day: "There is the problem. Seek its solution. You can find it by pure reason, for in mathematics there is no ignorabimus." (David Hilbert).

Good luck!

Problem 1		20
Problem 2		20
Problem 3		30
Problem 4		15
Problem 5		15
Extra Credit		20
Total		100

Problem 1 (20 points)

State the following results/definitions discussed in class. Do *not* prove anything.

(1) The AGM inequality.
(2) De Morgan's laws for logical operators.
(3) The definition of x^n (for n a natural number).
(4) The definition of being a finite set.
(5) The definition of the natural numbers.

Problem 2 (20 points)

Say whether the following statements are true or false, and *briefly* justify why.

(1) $\mathbb{Z}$ is equipotent to $\mathbb{Q}$.

Date: June 9, 2014.

(2) Every real number has a square root.
(3) If $p(x, y)$ is a propositional function, $\exists x \exists y\ p(x, y)$ is equivalent to $\exists y \exists x\ p(x, y)$.
(4) If x and y are rationals, then $x + y$ is rational.
(5) $p \to q$ is logically equivalent to $\neg q \vee p$.

Problem 3 (30 points)

It was seen in class that every natural number $n \geq 2$ can be written as a product of primes. Give the proof (you should give enough details to convince me that you understand the main ideas).

Problem 4 (15 points)

For sets A and B recall that the *symmetric difference* $A \Delta B$ of A and B is defined by $(A \backslash B) \cup (B \backslash A)$, where $A \backslash B$ is the set of elements of A that are not in B.
Show that for any sets A, B, C, $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.

Problem 5 (15 points)

Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$ for all real numbers x and y.
(1) Give three different examples of such a function.
(2) Prove that $f(0) = 0$. *Hint:* Write 0 as $0 + 0$.
(3) Prove that for any natural number n, $f(n) = nf(1)$.

Extra credit (20 points)

Assume $p(x)$, $q(x)$, and $r(x)$ are propositional functions. Assume you know that:
(1) $\forall x(p(x) \vee (q(x) \to r(x)))$
(2) $\neg \forall x(p(x) \vee r(x))$
Prove that $\neg \forall x q(x)$.