MATH 154 - PROBABILITY THEORY, SPRING 2018
ASSIGNMENT 1

Due Wednesday, January 31 at the beginning of class. Make sure to include your full name *and the list of your collaborators* (if any) with your assignment. You may discuss problems with others, but you may *not* keep a written record of your discussions. Please refer to the syllabus for details.

With regards to answering these problems, imagine that you are writing an answer to teach someone else in the class how to do the problem. In particular, you must give a complete outline for how you arrived at your answer. It is not sufficient to simply state a number or formula without providing the steps and reasoning that you used to produce the answer.

(1) Do problems 1, 2, 6 on p. 8 of Grimmett-Stirzaker.

(2) Do problems 1, 2, 7 on p. 12 of Grimmett-Stirzaker.

(3) Do problems 1, 2, 3, 6, 9 on p. 14 of Grimmett-Stirzaker.

(4) Let $(\Omega, \mathcal{F}, P)$ be a probability space. Suppose that for any $a \in \Omega$, $\{a\} \in \mathcal{F}$. Assume in addition that for any $a, b \in \Omega$, $P(\{a\}) = P(\{b\})$. Show that if Ω is infinite, then for any $a \in \Omega$, $P(\{a\}) = 0$. What if Ω is finite?

(5) *(Extra credit)* Let $\Omega := [0, 1]$ and let $\mathcal{F}$ be the σ-algebra of *all* subsets of Ω. Show that there is *no* probability measure on $(\Omega, \mathcal{F})$ that is translation invariant (a probability measure P is *translation invariant* if for any real number x and any $A \in \mathcal{F}$, if $A + x := \{a + x \mid a \in A\}$ is in $\mathcal{F}$, then $P(A) = P(A + x)$). *Hint*: Suppose for a contradiction that P is such a measure. Let E be the equivalence relation on $[0, 1]$ defined by xEy iff $x - y$ is rational. Let X be a subset of $[0, 1]$ containing exactly one representative of each E-equivalence class (called a *Vitali set*). Show that $P(X)$ cannot be zero but cannot be strictly positive either.

Date: January 24, 2018.