MATH 154 - PROBABILITY THEORY, SPRING 2018
ASSIGNMENT 11

Due Wednesday, April 18 at the beginning of class. Make sure to include your full name *and the list of your collaborators* (if any) with your assignment. You may discuss problems with others, but you may *not* keep a written record of your discussions. Please refer to the syllabus for details.

With regards to answering these problems, imagine that you are writing an answer to teach someone else in the class how to do the problem. In particular, you must give a complete outline for how you arrived at your answer. It is not sufficient to simply state a number or formula without providing the steps and reasoning that you used to produce the answer.

(1) Do problem 39 on p. 210 of Grimmett-Stirzaker.

(2) Do problem 48 on p. 211 of Grimmett-Stirzaker.

(3) Assume that a discrete time process $(X_n)_{n\in\mathbb{N}}$ with state space S satisfies the Markov condition (in the sense of Definition 6.1.1 in Grimmett-Stirzaker). Prove that for any $n \in \mathbb{N}$, any $n_1 < n_2 < \ldots < n_k \leq n$, and any $s, x_{n_1}, x_{n_2}, \ldots, x_{n_k} \in S$ such that $P(X_{n_1} = x_{n_1}, \ldots, X_{n_k} = x_{n_k}) > 0$, we have:

$$P(X_{n+1} = s | X_{n_1} = x_{n_1}, X_{n_2} = x_{n_2}, \ldots, X_{n_k} = x_{n_k}) = P(X_{n+1} = s | X_{n_k} = x_{n_k})$$

(4) Do problem 2 on p. 219 of Grimmett-Stirzaker.

(5) Do problem 1,2,3, and 4 on p. 223 of Grimmett-Stirzaker (a state i is *absorbing* if $p_{ii} = 1$; this means the chain will stay in state i with probability 1 once it has reached it).

Date: April 11, 2018.