MATH 154 - PROBABILITY THEORY, SPRING 2018
ASSIGNMENT 12

Due Friday, April 27, before 1pm. Make sure to include your full name *and the list of your collaborators* (if any) with your assignment. You may discuss problems with others, but you may *not* keep a written record of your discussions. Please refer to the syllabus for details.

With regards to answering these problems, imagine that you are writing an answer to teach someone else in the class how to do the problem. In particular, you must give a complete outline for how you arrived at your answer. It is not sufficient to simply state a number or formula without providing the steps and reasoning that you used to produce the answer.

(1) Let $(X_n)_{n\in\mathbb{N}}$ be a Markov chain and let p be the probability that we eventually enter a persistent state. That is:

$$p := P(\exists n : X_n \text{ is persistent})$$

(a) Give an example of a Markov chain where $p = 0$, an example where $p = 1$, and an example where $0 < p < 1$.

(b) Show that if S is finite, then $p = 1$.

(2) Do problem 1 on p. 225 of Grimmett-Stirzaker. There, $r \in [0, 1]$, and $(a_n)_{n\in\mathbb{N}}$ is a nonincreasing sequence of real numbers such that $\sum_n a_n = 1$. "Classify the states" means finding out for each i whether state i is transient, null persisent, or non-null persistent. This may of course depend on r and on the a_i's.

(3) Do problems 4(a) and 4(b) on p. 226 of Grimmett-Stirzaker.

(4) Do problems 1, 6, and 8 on p. 236 of Grimmett-Stirzaker.

Date: April 23, 2018.