MATH 154 - PROBABILITY THEORY, SPRING 2018
ASSIGNMENT 3

Due Wednesday, February 14 at the beginning of class. Make sure to include your full name *and the list of your collaborators* (if any) with your assignment. You may discuss problems with others, but you may *not* keep a written record of your discussions. Please refer to the syllabus for details.

With regards to answering these problems, imagine that you are writing an answer to teach someone else in the class how to do the problem. In particular, you must give a complete outline for how you arrived at your answer. It is not sufficient to simply state a number or formula without providing the steps and reasoning that you used to produce the answer.

(1) Do problem 1 on p. 38 of Grimmett-Stirzaker.
(2) Do problems 1 and 4 on p. 41 of Grimmett-Stirzaker.
(3) Do problems 1 and 6 on p. 43 of Grimmett-Stirzaker.
(4) Do problem 7 on p. 44 of Grimmett-Stirzaker.
(5) Do problem 2 on p. 49 of Grimmett-Stirzaker.
(6) Do problem 8 on p. 55 of Grimmett-Stirzaker.
(7) Let X and Y be random variables. Let F_X be the joint distribution function of X and let $F_{X,Y}$ be the joint distribution of (X, Y). Prove that for any real number x, $F_X(x) = \lim_{y\to\infty} F_{X,Y}(x, y)$.
(8) Let $(\Omega, \mathcal{F}, P)$ be a probability space. Let X be a *discrete* random variable and let S be a subset of $\mathbb{R}$. Show that the set $\{X \in S\} = \{\omega \in \Omega \mid X(\omega) \in S\}$ is in $\mathcal{F}$. Deduce that if $g : \mathbb{R} \to \mathbb{R}$ is a function, then $g(X)$ is a discrete random variable.
(9) Let X and Y be discrete random variables and let $g, h : \mathbb{R} \to \mathbb{R}$ be functions. Show the following:
    (a) If X and Y are independent, then $g(X)$ and $h(Y)$ are also independent.
    (b) $E(g(X)) = \sum_x g(x) f_X(x)$, whenever the sum is absolutely convergent.

Date: February 7, 2018.